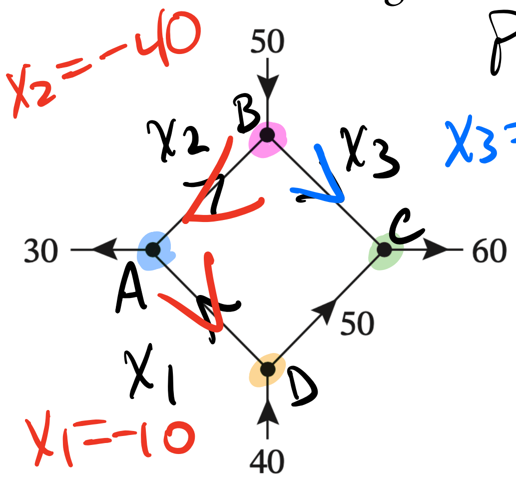
1.10 – Applications of Linear Systems

Definitions: A **network** is a set of branches through which something flows. Branches meet at **nodes** or **junctions**. Networks in our work will feature **flow conservation**, meaning the rate of flow in and out are equal at each node.

#**1** The accompanying figure shows a network in which the flow rate and direction of flow in certain branches are known. Find the flow rates and directions of flow in the remaining branches.



Pick a direction

Rate in = Rate out

Node A: $x_1 = 30 + x_2$

Node B: $x_2 + 50 = x_3$

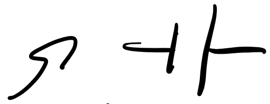
Node C: $x_3 + 50 = 60$

Node D: $40 = x_1 + 50$

$x_1 = -10$

$x_2 = -40$

$x_3 = 10$

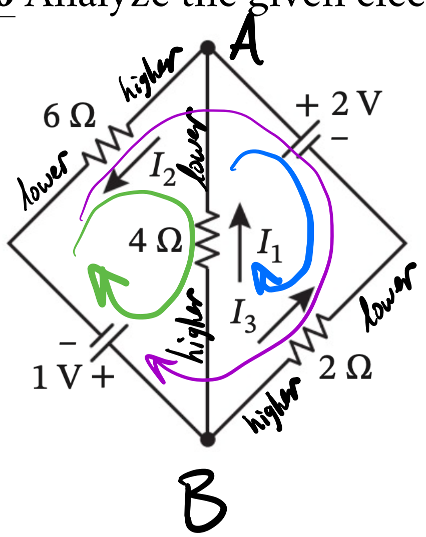


Definitions: A **battery** is a source of electric energy, and a **resistor** dissipates electric energy. A **node** is where three or more wires join in a circuit. A **closed loop** begins and ends at the same node.

Ohm's Law: $E = IR$, where E (volts) is voltage drop at a resistor with resistance R (ohms) in a circuit with current I (amperes).

Kirchhoff's Laws (summary): Net current (in and out) at a node is zero, and net voltage change (rises and drops) in a closed loop is zero.

#6 Analyze the given electrical circuit by finding the unknown currents.



Node A:

in out

$I_1 + I_3 - I_2 = 0$

Node B:

$I_2 - I_1 - I_3 = 0$

equivalent

Blue clockwise loop:

voltage rises voltage drops

$2I_3 - 4I_1 - 2 = 0$

Green loop: $6I_2 + 4I_1 - 1 = 0$

Purple loop: $-2 + 2I_3 - 1 + 6I_2 = 0$

Rewrite:

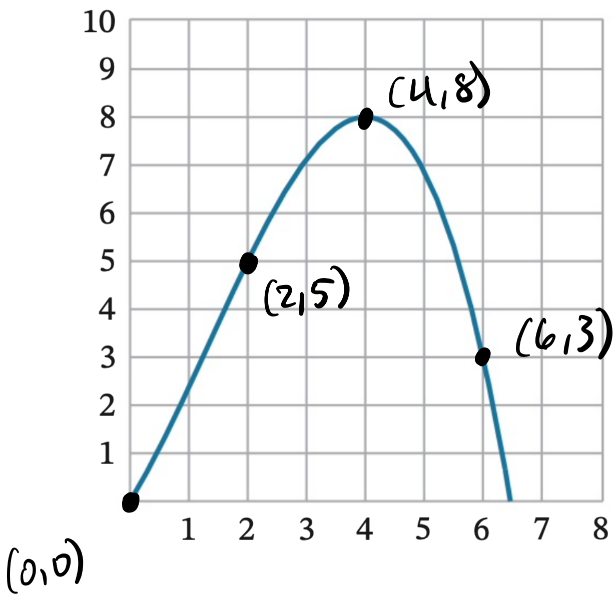
$$I_1 - I_2 + I_3 = 0$$
$$-4I_1 \quad + 2I_3 = 2$$
$$4I_1 + 6I_2 \quad = 1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -4 & 0 & 2 & 2 \\ 4 & 6 & 0 & 1 \end{array}\right] \rightarrow \boxed{\begin{array}{l} I_1 = -5/22 \text{ A} \\ I_2 = 7/22 \text{ A} \\ I_3 = 6/11 \text{ A} \end{array}}$$

Theorem 1.10.1 Polynomial Interpolation

Given any n points in the xy-plane that have distinct x-coordinates, there is a unique polynomial of degree $n-1$ or less whose graph passes through those points.

#16 The accompanying figure shows the graph of a cubic polynomial. Find the polynomial.



$$y = ax^3+bx^2+cx+d$$

$(2,5)$ $\quad 5 = 8a+4b+2c+d$

$(4,8)$ $\quad 8 = 64a+16b+4c+d$

Aug. matrix:
$$\begin{array}{c} \begin{array}{cccc} a & b & c & d \ \text{Const} \end{array} \\ \left[\begin{array}{cccc|c} 8 & 4 & 2 & 1 & 5 \\ 64 & 16 & 4 & 1 & 8 \\ \multicolumn{4}{c}{\text{and so on}} & \end{array}\right] \end{array}$$

use $d+cx+bx^2+cx^3=y$

$$\left[\begin{array}{cccc|c} 1 & 2 & 4 & 8 & 5 \\ 1 & 4 & 16 & 64 & 8 \\ \multicolumn{4}{c}{\text{and so on}} & \end{array}\right]$$

So much nicer